Új Nemzeti
Kiválóság Program

II. Properties of the nucleus

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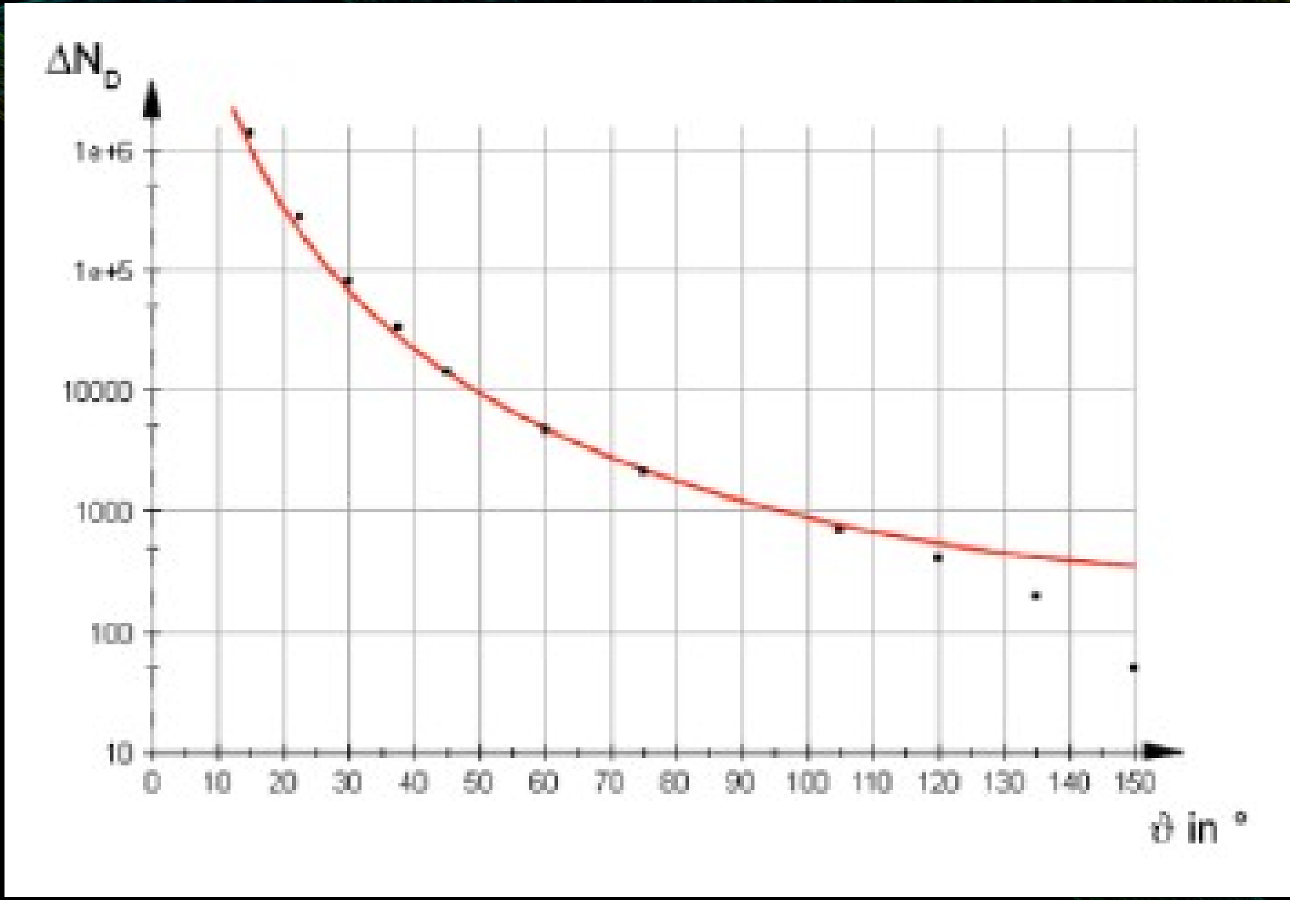
Nuclear radii and densities I.

- Rutherford formula is not satisfactory at large angles (small impact parameters)
- Deviation is due to finate nuclear size → strong force beside Coulomb (increasing with energy)



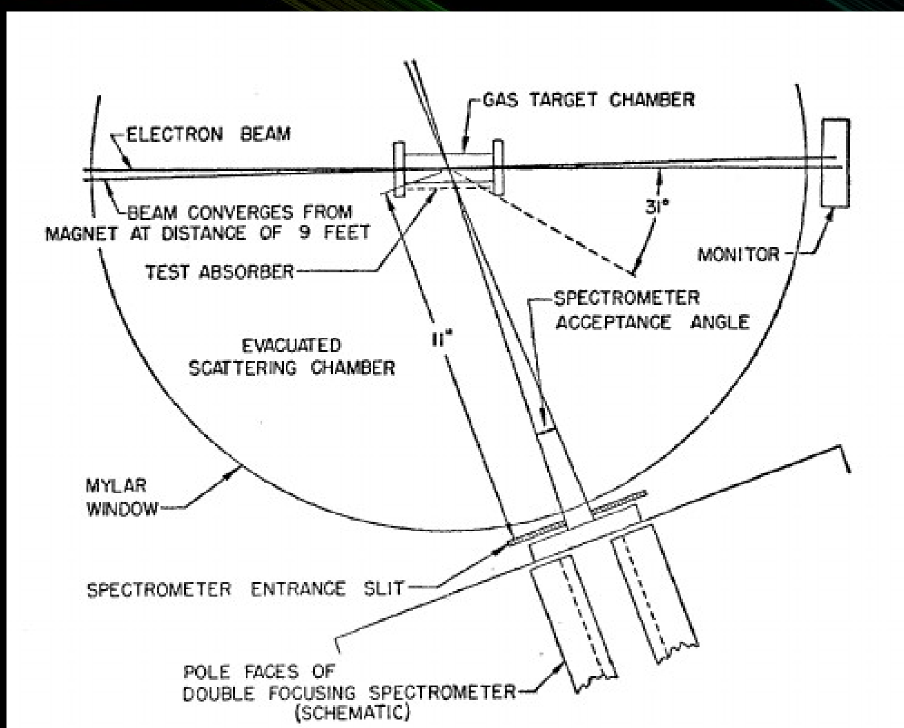
Nuclear radii and densities II. - e⁻ scattering

- R. Hofstadter: measuring the charge distribution with fast (100-500 MeV - relativistic) electron elastic scattering

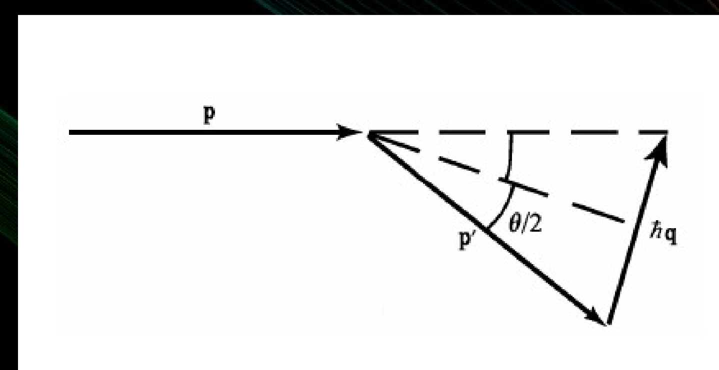
$$E=\sqrt{(pc)^2+(m_0c^2)^2}\simeq pc \quad \longrightarrow \quad \lambda\!\!\!^{-}=\frac{\hbar}{p}=\frac{\hbar c}{E}\approx\frac{200}{E(MeV)}fm$$

resolution

- Elastic scattering: p’=p, the direction may change only → **Δp** = ħ***q***



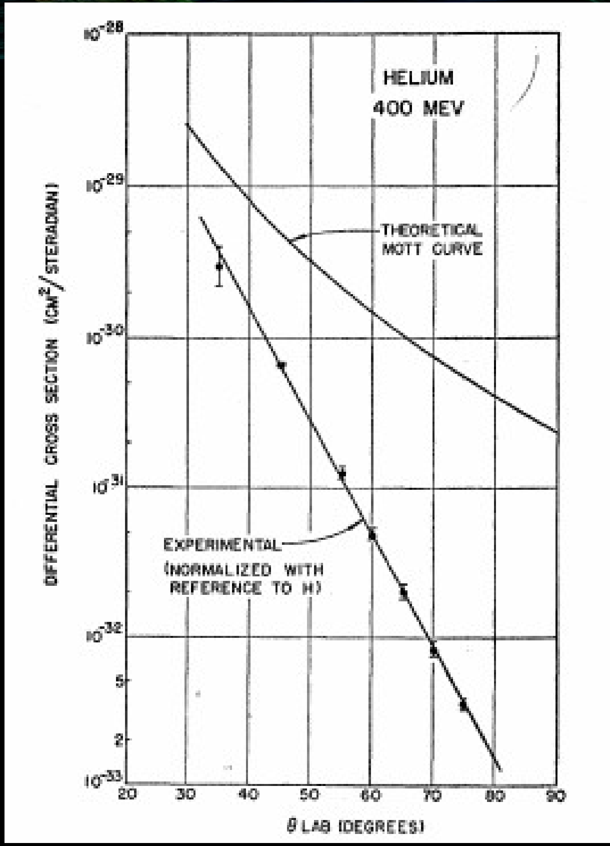
$$q=\frac{2E}{\hbar c}\sin\left(\frac{\phi}{2}\right)$$

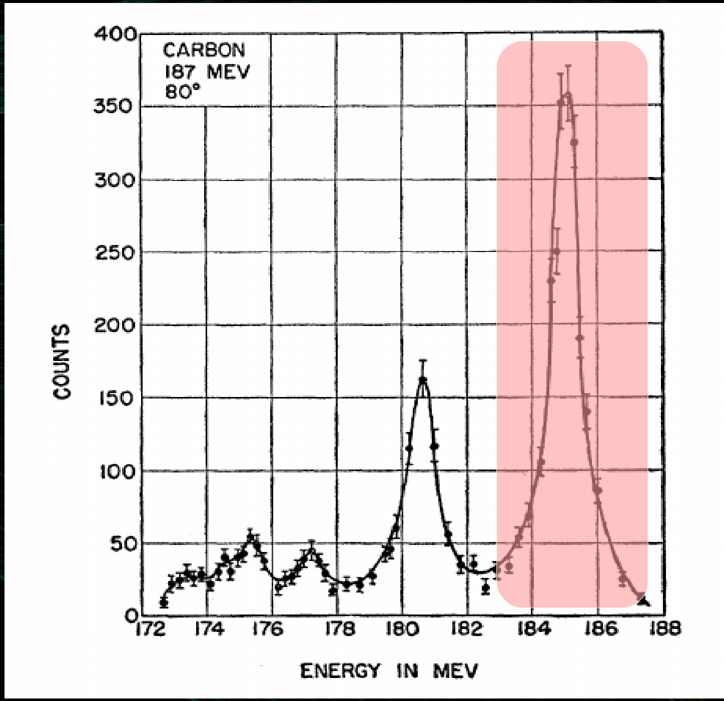


Nuclear radii and densities III. - Mott formula

- Mott: cross section for point-like nucleus

$$\sigma_M(\vartheta) \simeq \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}}$$





- For finite-size nucleus: form factor ***F(q)***

$$\sigma(\phi) = \sigma_M(\phi) |F(\vec{q})|^2$$

- In case of ρ(r) charge density with spherical symmetry:

$$F(q) = \int \rho(r) \frac{\sin(qr)}{qr} dv$$

Nuclear radii and densities V.

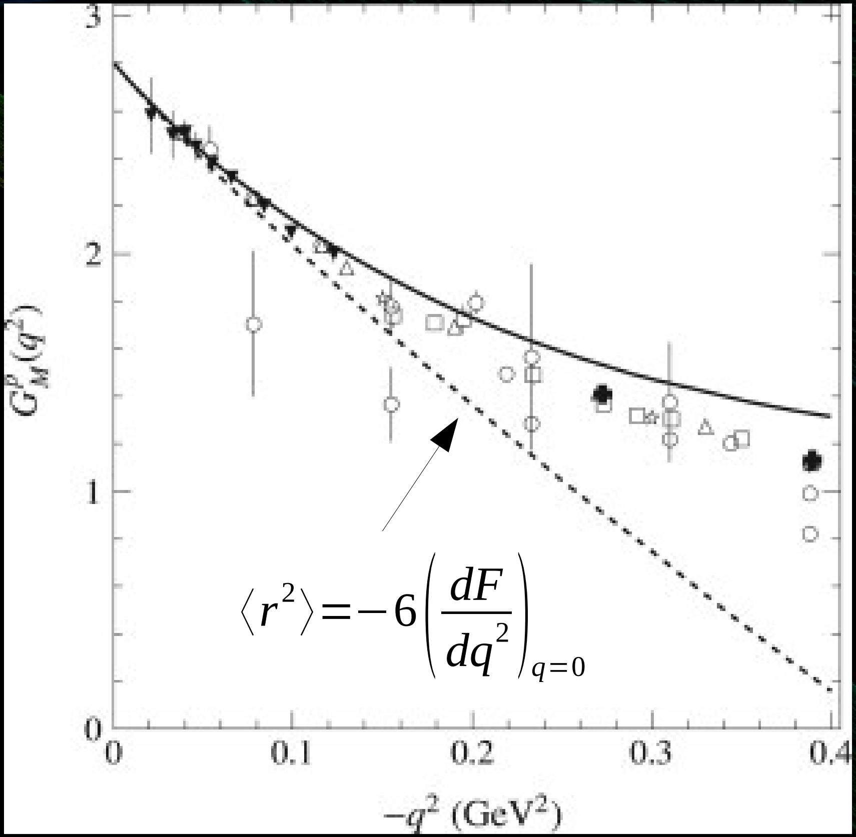
- E<100 MeV: q<<1/R → qr<<1

$$F(q)=\int q(r)\frac{qr-\frac{(qr)^3}{3!}+\ldots}{qr}dv=1-\frac{q^2\langle r^2\rangle}{6}+\ldots$$

mean quare radii
(root is **rms** radii)

$$\langle r^2\rangle\equiv\int\rho(r)r^2dv$$

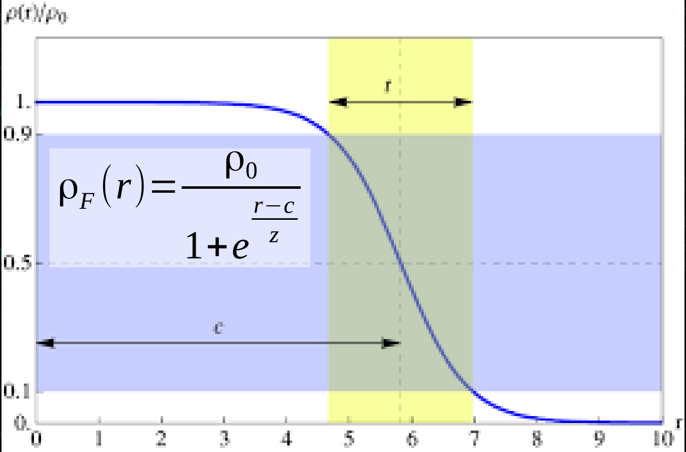
Model-
independent
!!



$$\langle r^2\rangle=-6\left(\frac{dF}{dq^2}\right)_{q=0}$$

Nuclear radii and densities – VI.

- E>100 MeV: approximation is no longer satisfactory (senitive to nuclear charge distribution) → using model functions to describe the shape of the charge distribution (large λ, good resolution allows!!) → fitting the function to extract the parameters
- e.g Fermi function : no sharpe edge of nucleus



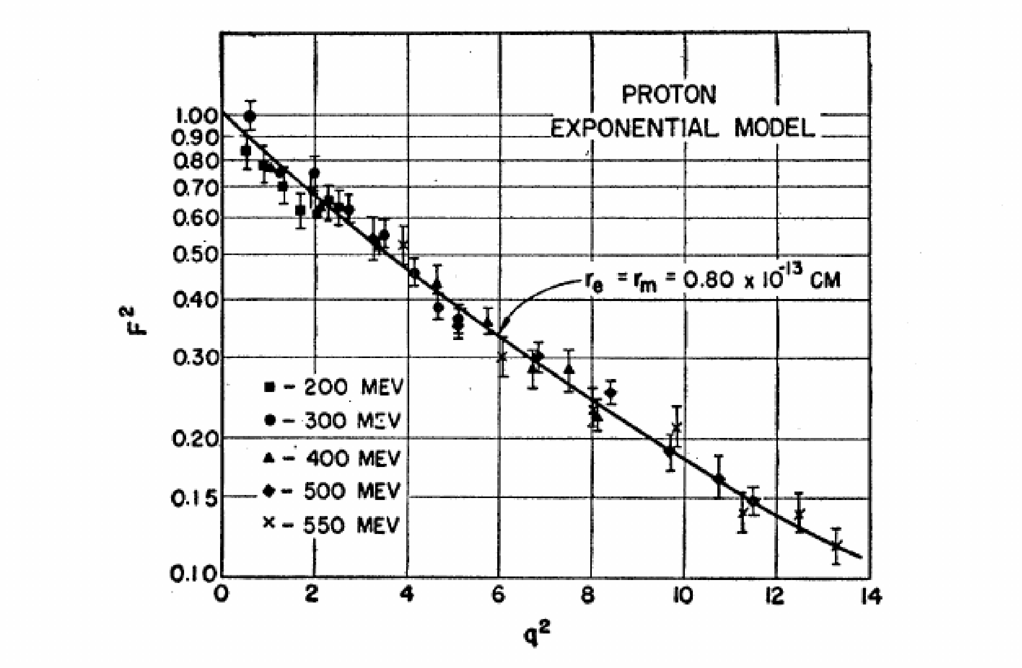
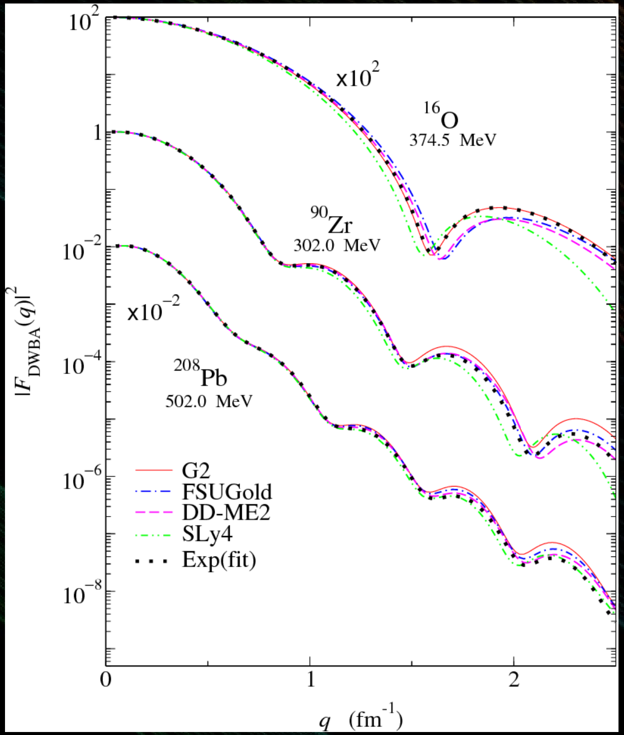


FIG. 27. The square of the form factor plotted against q^2. q^2 is given in units of 10^{-26} cm². The solid line is calculated for the exponential model with rms radii $=0.80\times10^{-13}$ cm.



Nuclear radii and densites IV. - Form factors

TABLE I. In this table $\rho(r)$ is the charge density function; "a" is the root-mean-square radius of the charge distribution; $F(qa)$ is the form factor; $x=qa$.

Model number	Name of model	Expression for charge density $4\pi a^3\rho(r)$; $y=r/a$	$F(qa)$; $x=qa$
I	Point	δ function	1
II	Uniform	$\begin{cases} \frac{9}{5}\left(\frac{3}{5}\right)^{\frac{1}{2}} \text{ for } y\leqslant\left(\frac{5}{3}\right)^{\frac{1}{2}} \\ 0 \text{ for } y\geqslant\left(\frac{5}{3}\right)^{\frac{1}{2}} \end{cases}$	$5\left(\frac{5}{3}\right)^{\frac{1}{2}} x^{-3}\left[\sin\left(\frac{5}{3}\right)^{\frac{1}{2}} x-\left(\frac{5}{3}\right)^{\frac{1}{2}} x\cos\left(\frac{5}{3}\right)^{\frac{1}{2}} x\right]$
III	Gaussian	$3\left(\frac{6}{\pi}\right)^{\frac{1}{2}}\exp\left(-\frac{3}{2}y^2\right)$	$\exp(-x^2/6)$
IV	Exponential	$12\sqrt{3}\exp(-(12)^{\frac{1}{2}}y)$	$\left(1+\frac{x^2}{12}\right)^{-2}$
V	Shell	$\delta(y-1)$	$x^{-1}\sin x$
VI	Hollow exponential	$\frac{200}{3}y\exp(-(20)^{\frac{1}{2}}y)$	$\left(1-\frac{x^2}{60}\right)\left(1+\frac{x^2}{20}\right)^{-3}$
VII	...	$\frac{75}{2}(30)^{\frac{1}{2}}y^2\exp(-(30)^{\frac{1}{2}}y)$	$\left(1-\frac{x^2}{30}\right)\left(1+\frac{x^2}{30}\right)^{-4}$
VIII	Yukawa I	$\sqrt{2}y^{-2}\exp(-\sqrt{2}y)$	$\sqrt{2}x^{-1}\tan^{-1}(x/\sqrt{2})$
IX	Yukawa II	$6y^{-1}\exp(-\sqrt{6}\,y)$	$\left(1+\frac{x^2}{6}\right)^{-1}$
X	Hollow Gaussian	$\frac{50}{3}\left(\frac{5}{2\pi}\right)^{\frac{1}{2}}y^2\exp\left(-\frac{5}{2}y^2\right)$	$\left(1-\frac{x^2}{15}\right)\exp\left(-\frac{x^2}{10}\right)$
XI	Generalized shell model	$\begin{cases} \frac{8}{\sqrt{\pi}}\frac{k^3}{(2+3\alpha)}(1+\alpha k^2y^2)\exp(-k^2y^2) \\ \text{where } k=\left[\frac{3(2+5\alpha)}{2(2+3\alpha)}\right]^{\frac{1}{2}} \end{cases}$	$\left[1-\frac{\alpha x^2}{2k^2(2+3\alpha)}\right]\exp\left(-\frac{x^2}{4k^2}\right)$
XII	Modified exponential	$\frac{27}{\sqrt{2}}[1+(18)^{\frac{1}{2}}y]\exp[-(18)^{\frac{1}{2}}y]$	$\left(1+\frac{x^2}{18}\right)^{-3}$

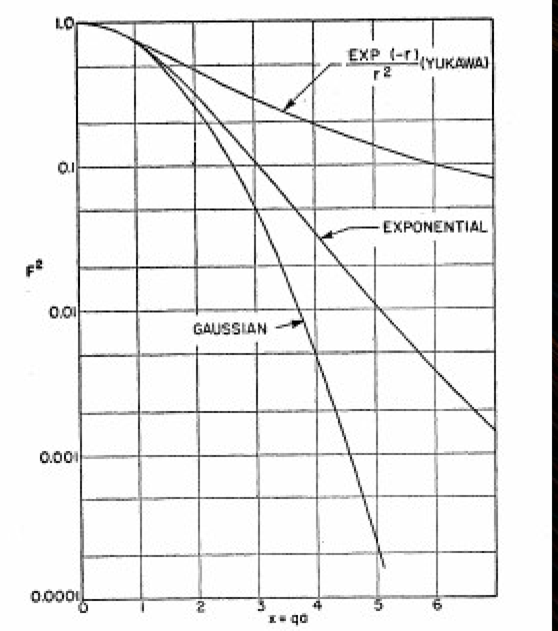
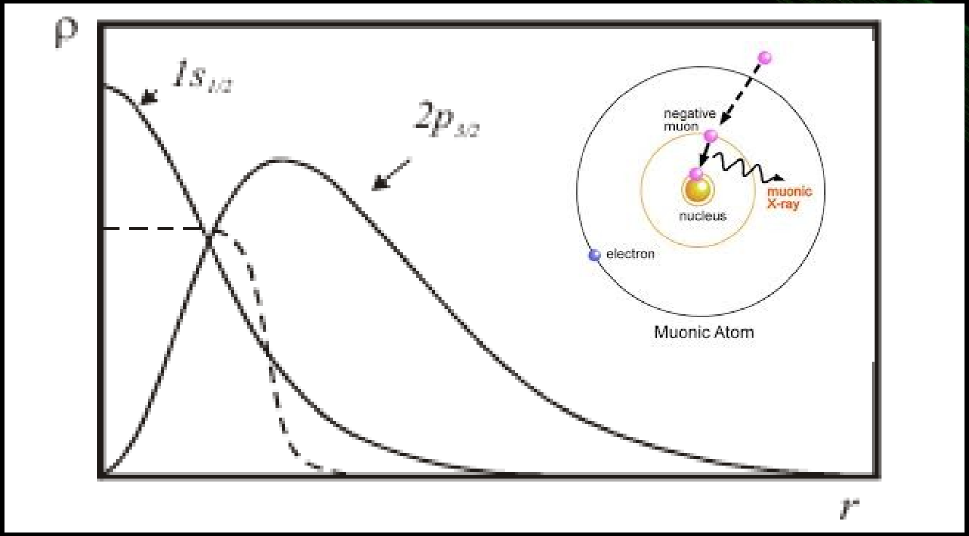


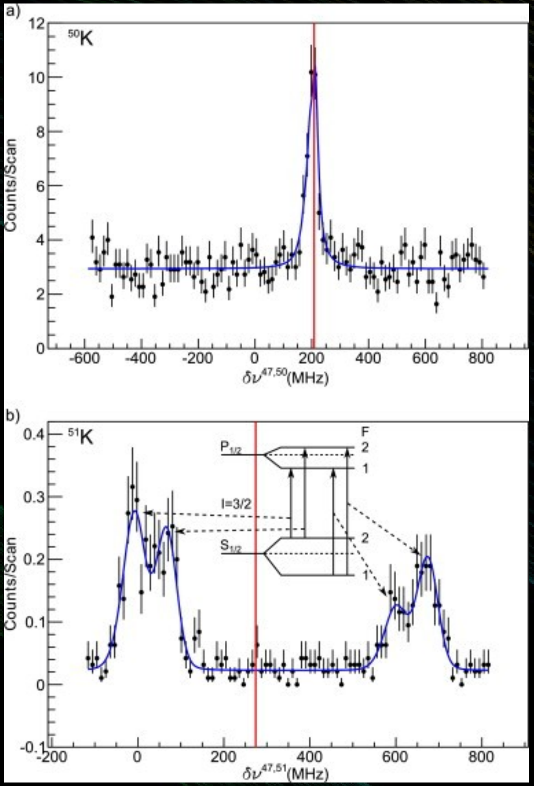
FIG. 3. The square of the form factor for typical charge distributions.

Nuclear radii and denstities VII.

- **Characteristic X-ray spectroscopy of muonic atoms**
  - $m_\mu = 207 m_e$
  - Bohr radius is 1/207 smaller (at given *n*)
  - Wavefunction of nucleus and $1s_{1/2}$ is overlapping → energy is depending on the size of nucleus → $\langle r^2 \rangle$
  - Fitch and Rainwater (1953)

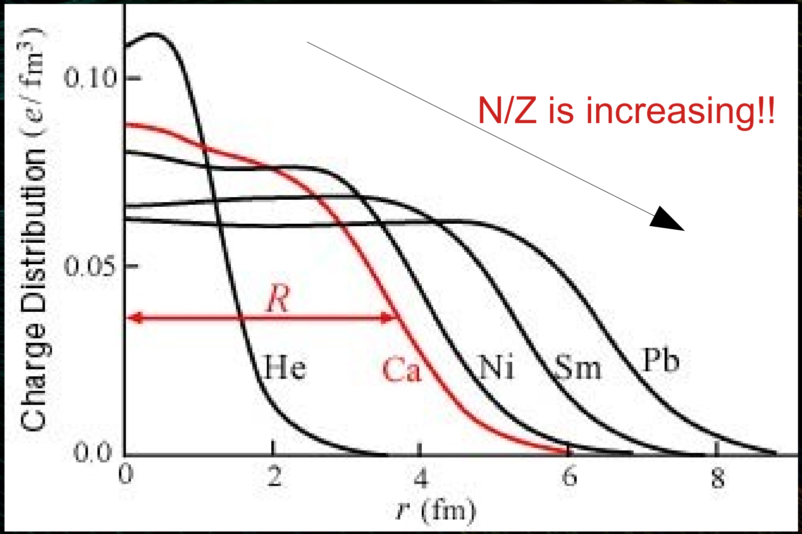


- **Optical isotope shift**:
  - only technique for radiactive nuclei
  - $\Delta\nu_{12} \rightarrow \delta_{12}\langle r^2 \rangle$
  - relative measurement



Nuclear radii and density: results

- No sharp endges of nuclei: charge density is dropping „smoothly”

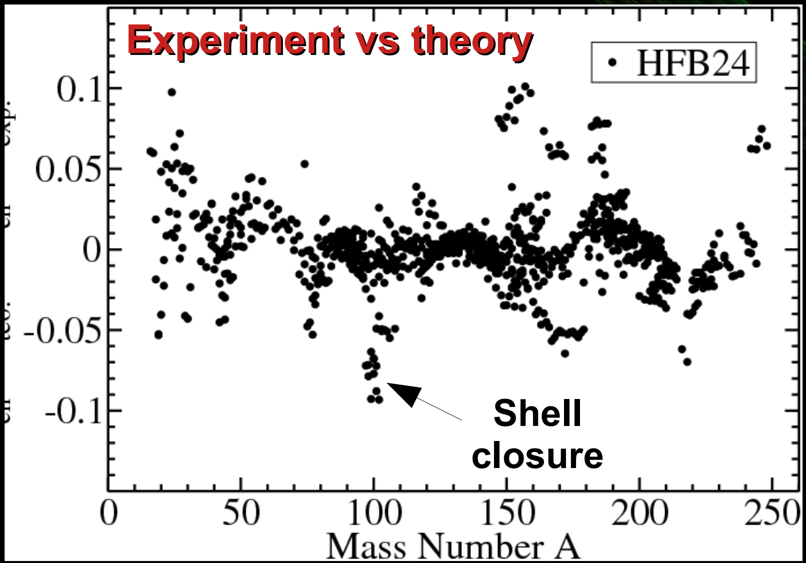


$$R(N,Z)=\sqrt{\langle r^2\rangle}$$

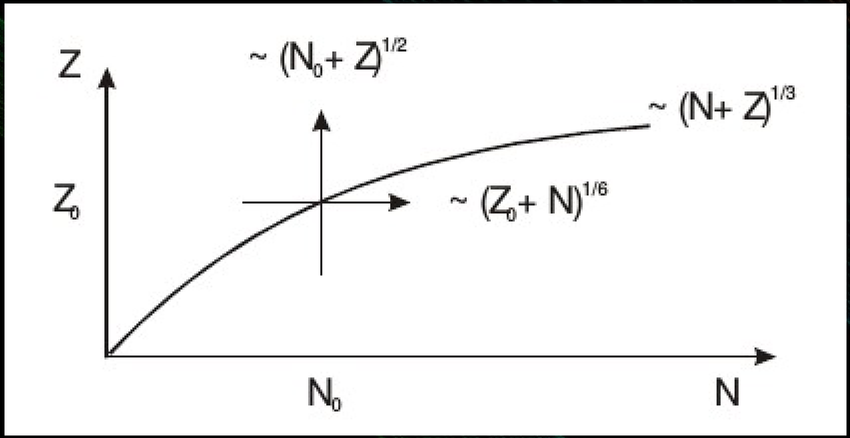
RMS radii

- Stable isotopes

$$R_{st}=r_0 A_{st}^{1/3}, where\ r_0 \simeq 0.95\ fm$$



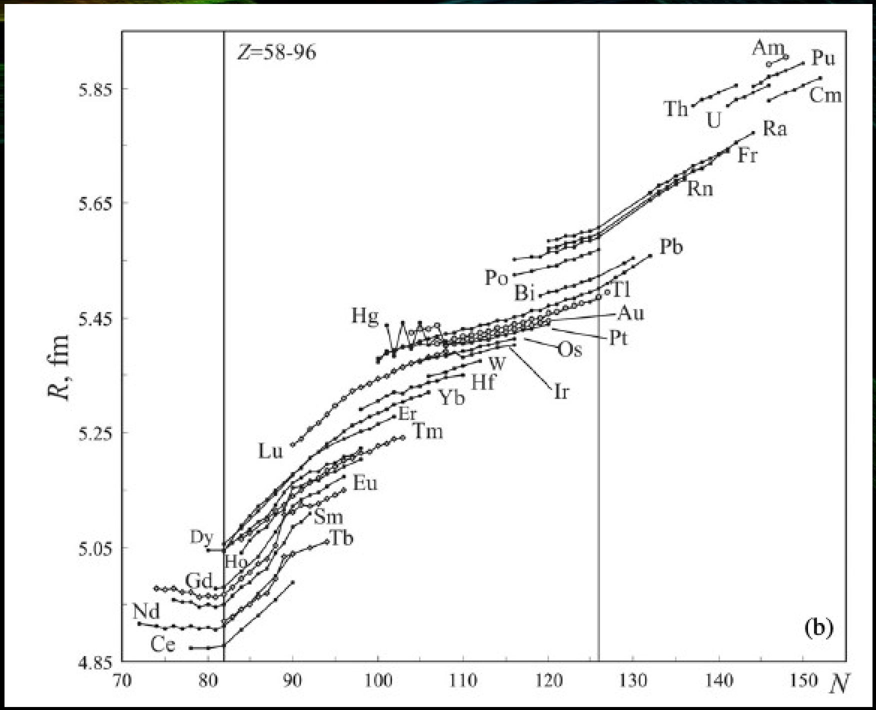
- Non-stable isotopes



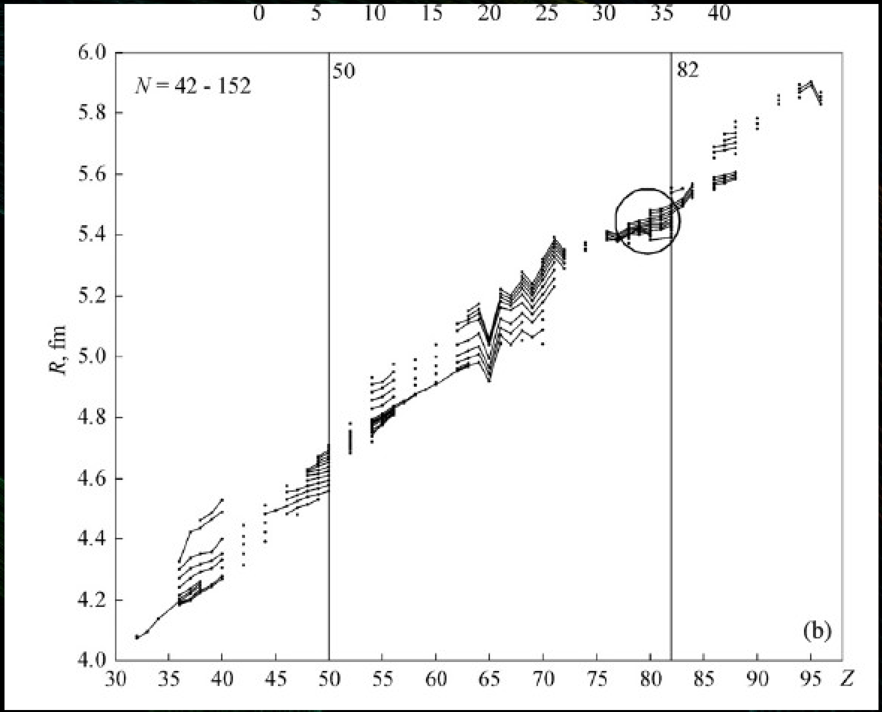
Nuclear radii and density: results

- most recent compilation for non-stable nuclei

Isotopic dependence



Isotonic dependence



Mass of nucleus I.

$$1\,amu = \frac{1}{12} m_{^{12}C}$$

- Magnetic mass spectrometers
  - Dempster (1918): direction focusing

Acceleration

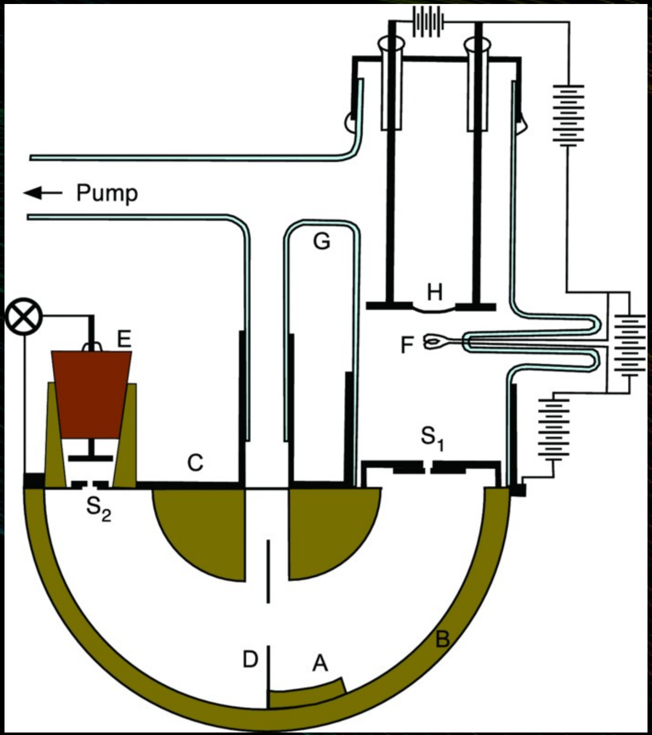
$$eV = \frac{1}{2} Mv^2$$

Lorentz-force

$$evB = \frac{Mv^2}{R}$$

(B constant)

$$M = \frac{R^2}{V} \frac{eB^2}{2}$$



- Fixed V: magnetic spectrograph

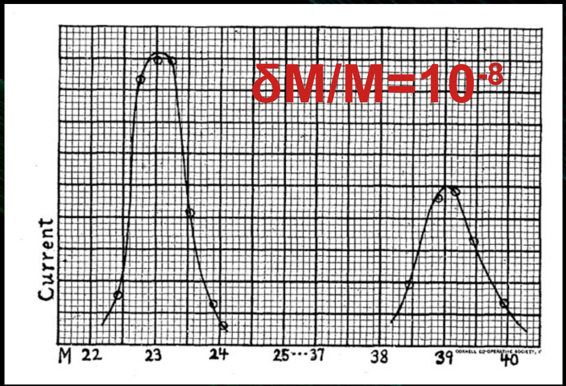
$$\frac{M_1}{M_2} = \frac{R_1^2}{R_2^2}$$

Relative measurements

- Fixed R: magnetic spectrometer

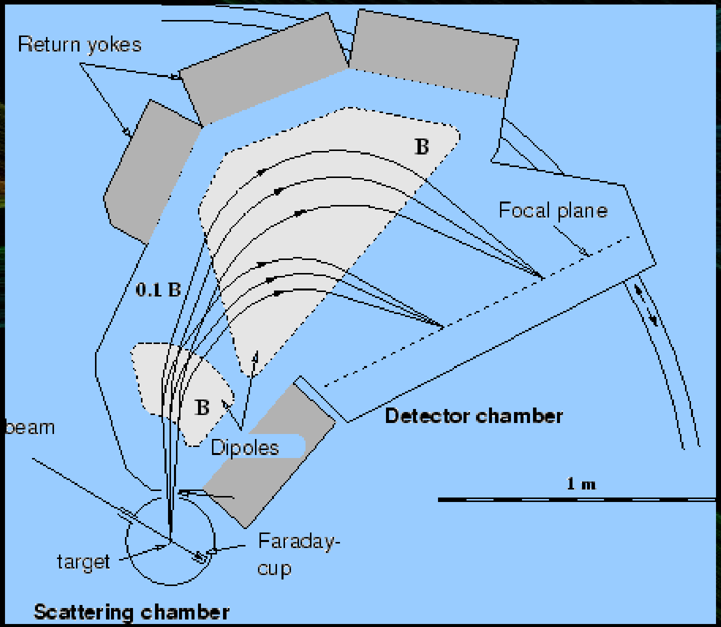
$$\frac{M_1}{M_2} = \frac{V_2}{V_1}$$

Relative measurements

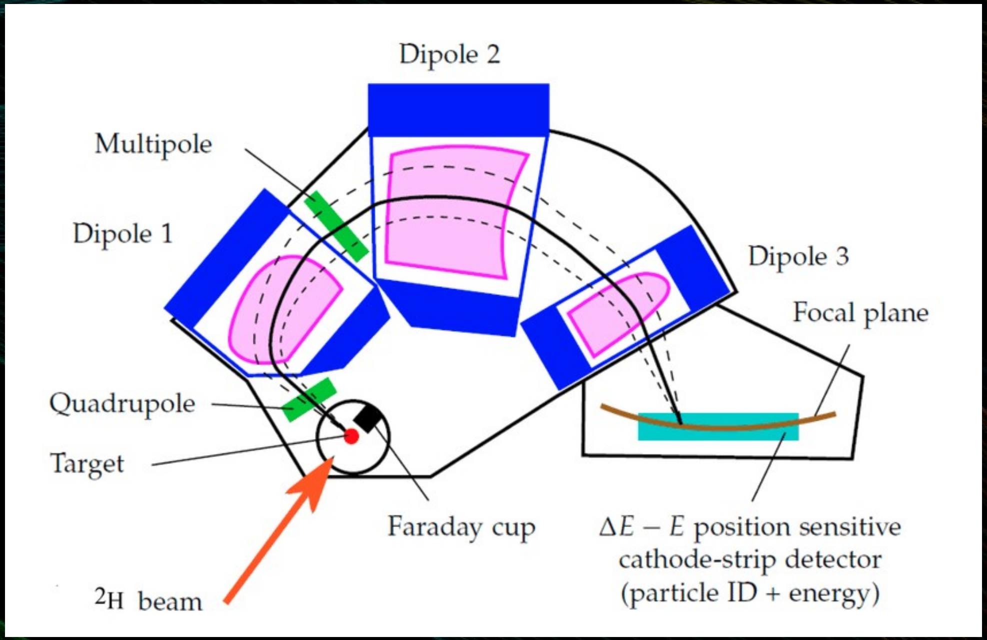


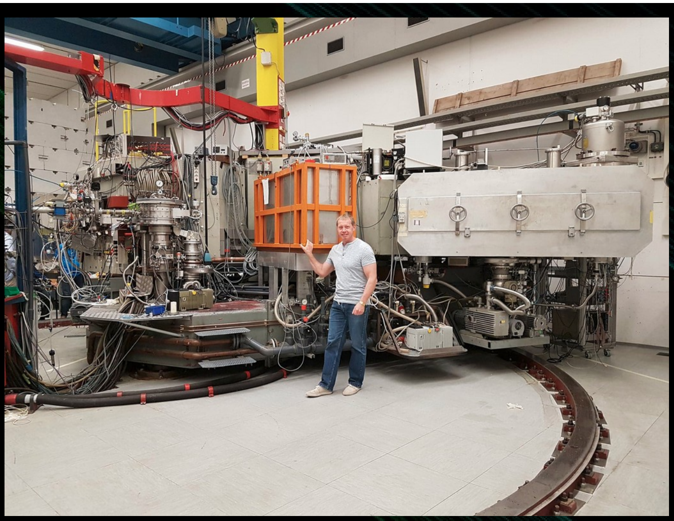
Magnetic spectrographs: high resolution spectroscopy today

- Split-pole spectrograph at Debrecen



- Q3D spectrograph at Munich



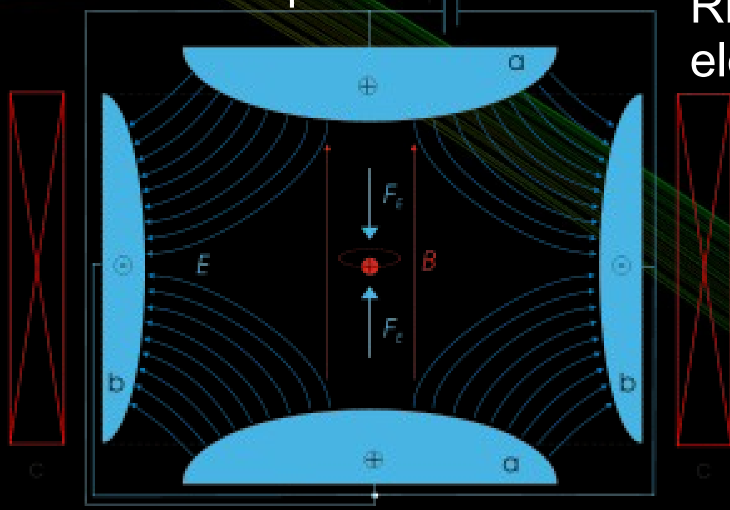


Mass of nuclei II.

- Penning trap
  - cyclotron motion: Lorentz motion by a B magnetic field x velocity (ω_c)
  - axial motion: vertical component of E quadrupole electrostatic field (ω_a)
  - magnetron motion: horizontal component of E field (ω_m)

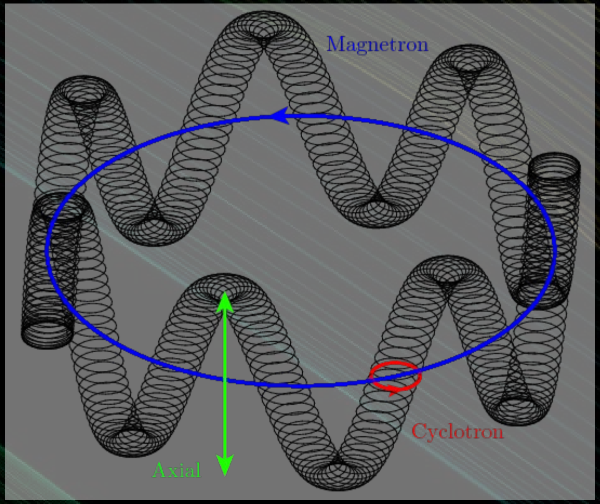
Top electrode

Ring electrode



Bottom electrode

$\omega_c >> \omega_a >> \omega_m$



$$\frac{M_1}{M_2} = \frac{\omega_{c1}}{\omega_{c2}}$$

$\delta M/M = 10^{-10}$

- Good for radioacive isotopes!!!

Mass of nuclei III.

- Neutral nuclei: e.g. mass of neutron? (magnetic spectrometers are not good)
- Solution: energy balance of nuclear reactions

$$A + B \rightarrow C + D$$

$$(M_A+M_B)c^2+E_A+E_B=(M_C+M_D)c^2+E_C+E_D$$

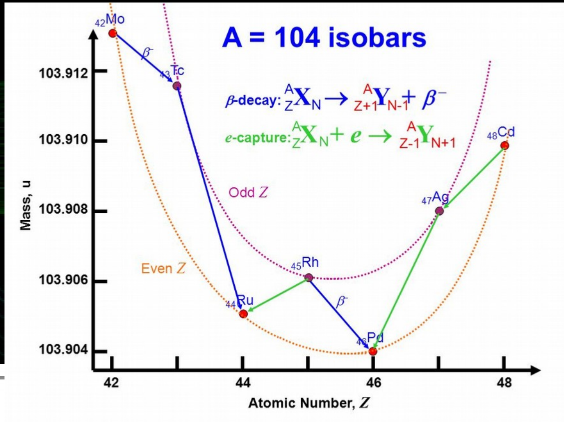
$p + n \rightarrow d + \gamma$ the neutron mass can be extracted

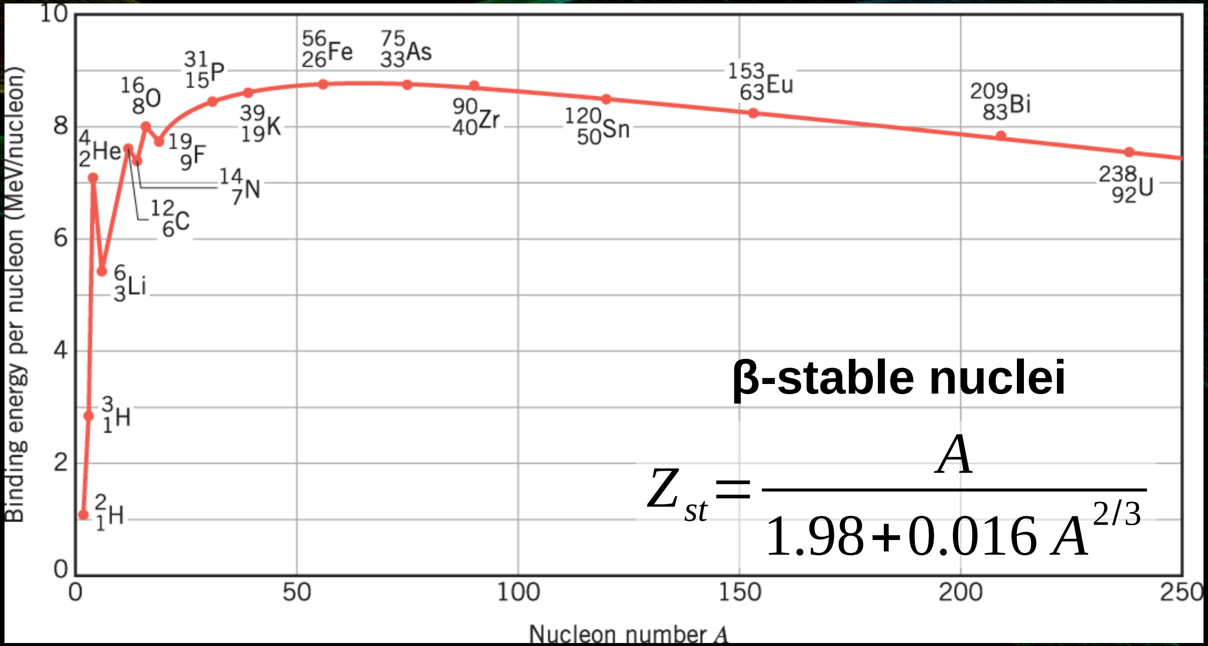
M_p = 938.3 MeV = 1836.1 x m_e M_n = 939.6 MeV = 1838.6 x m_e

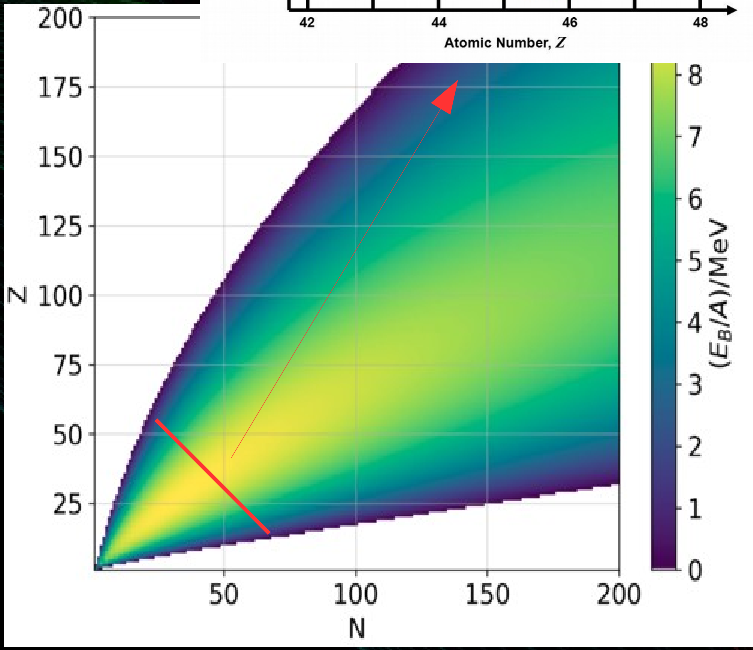
- energy balance of alpha and beta decay: dE_α=5 keV → $\delta M/M=10^{-8}$
- microwave spectroscopy of rotational levels of molecules

Mass of nuclei IV.: binding energy

- Binding energy from masses: $E_k(Z,A)=[ZM_p+(A-Z)M_n-M(Z,A)]c^2$
- Experimental findings → **$E_k \sim A$**
- Binding energy/nucleon → $\varepsilon(Z,A)=E_k/A \approx 8$ MeV/nucleon





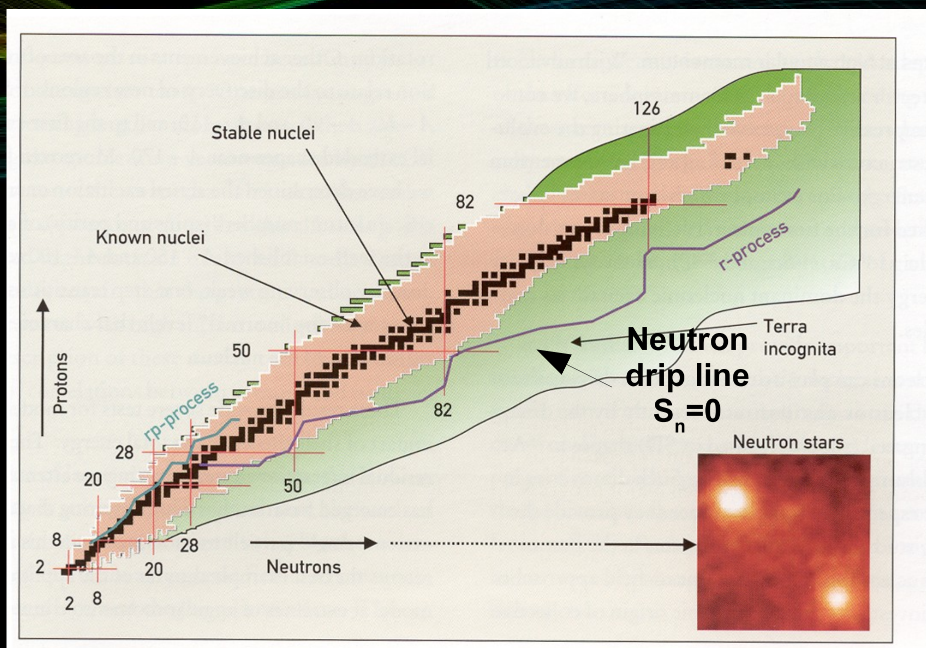


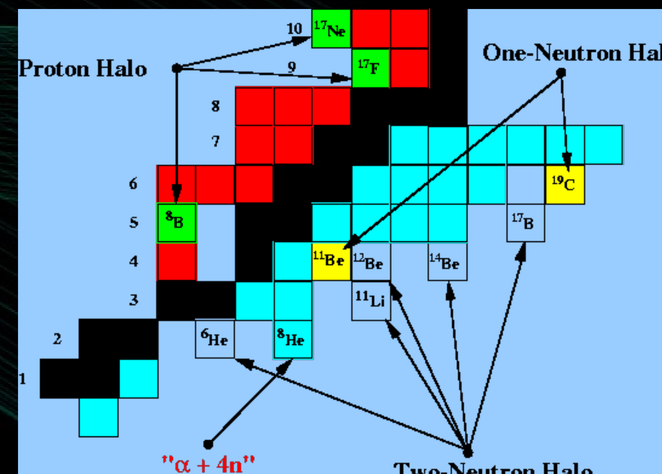
- binding energy is saturated → short range nuclear force
- fission and fusion

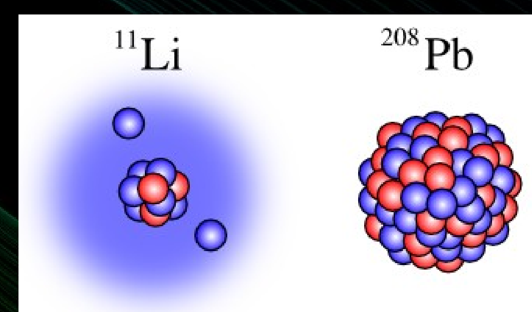
- More properties:
  - pairing effect (only 4 stable odd-odd nuclei!!)
  - Magic numbers: Z or/and N =2,8,20,28,50,82,126 → nuclear shell model

The nuclear landscape

- Nucleon-stablity: nucleon separation energy is positive (S_n>0 and S_p>0)
  - 6-7000 nuclei (known ~2830, β-stable ~280)



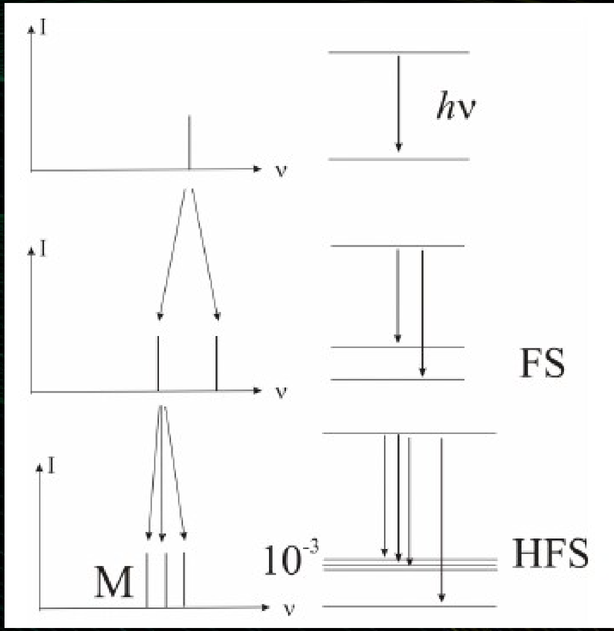




- Exotic nuclear radii and densities near the drip lines: *halo*-nuclei!

Nuclear moments: spin and magnetic moment

- *Fine structure* of spectral lines → spin and magnetic moment of e^-
- *Hyperfine structure* of spectral lines → spin and magnetic moment of nucleus



Brief overview on the e^- spin and magnetic moment

- Interaction of the magnetic moment of e- and the magnetic field by the e- motion in atoms
- Given *s* → 2s + 1 different *U* value
- Only valence electrons attribute to μ_s → 1 valence electron → doublet spectral line

$$U = -\vec{\mu}_s \vec{H}_e$$

$$\mu_s = \frac{e\hbar}{2 m_e c}$$

$$s = \frac{1}{2}\hbar$$

Nuclear moments: spin and magnetic moment

- Pauli (1928) hyperfine structure of spectral lines → nucleus has spin and magnetic moment!
- **Spin**: $|\vec{I}| = I(I+1)\hbar^2$ $\quad (\vec{I})_z = m\hbar \quad (m = I, I-1, \ldots, -I)$

  I is non-negative integer or half-integer

  multiplicity: *M=2I+1* different *m* values

- **Magnetic dipole momentum:**

$$g = \frac{\mu/\mu_p}{I}$$ g-factor

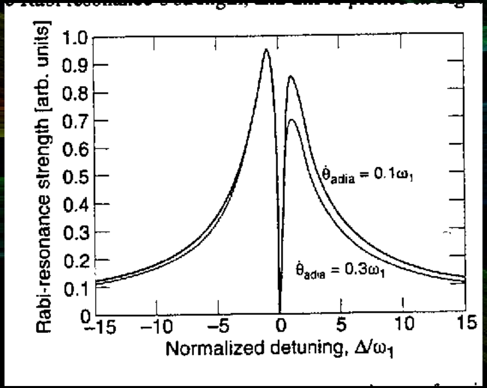
$$\mu_p = \frac{e\hbar}{2m_p c} = \frac{m_e}{m_p} M_B$$ magnetic moment of proton

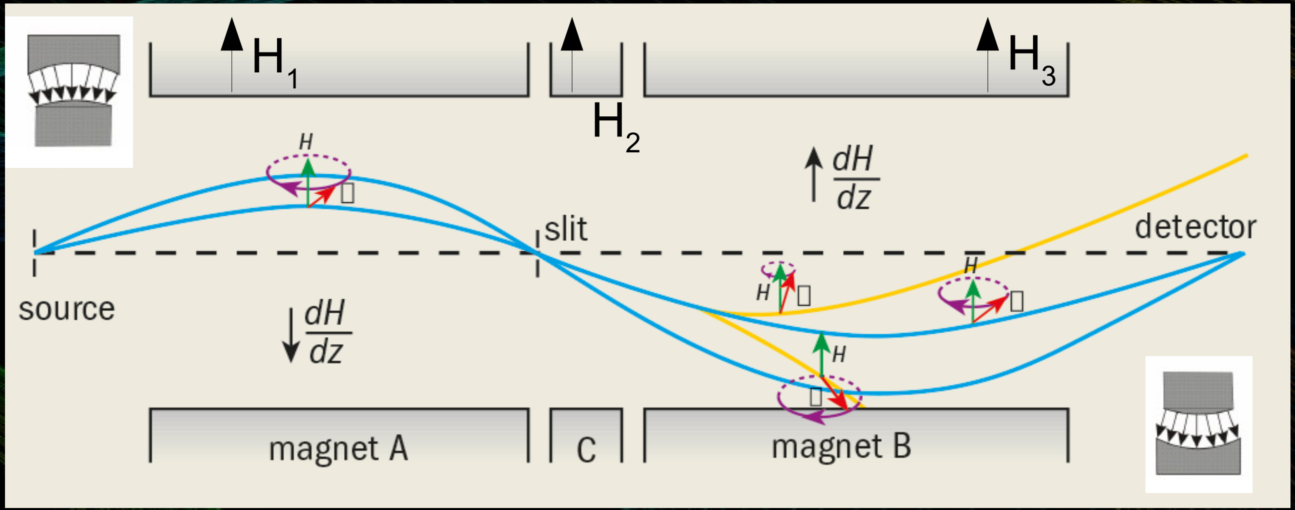
$$\vec{H}_e = -a\frac{\vec{J}}{|\vec{J}|} \;\Rightarrow\; \vec{\mu} = \mu\frac{\vec{I}}{|\vec{I}|} \qquad U = -\vec{\mu}\,\vec{H}_e = \mu a \frac{\vec{J}\,\vec{I}}{|\vec{J}||\vec{I}|}$$

- If *J>I* counting spectral lines → multiplicity *M* from spectrum (different *m* → different energy) → *I*

Nuclear moments: magnetic dipole

- Rabi (1939)





- Results:
  - $s_p = 1/2\hbar$ and $s_n = 1/2\hbar$
  - anomal magnetic moment: $\mu_p = 2.79\mu_N$ and $\mu_n = -1.91\mu_N$
  - ^2H: $s = 1\hbar$ and $\mu = 0.86\mu_N$ → spins are parallel → nuclear force is *spin dependent*
  - Even-even nuclei → spins and magnetic moments are zero!! (in ground state) → *pairing effect* is important

Nuclear moments: electric quadupole moment

- Due to mirror symmetry → *no electric dipole* (and any odd order) momentum!
- Deviations in the hyperfine structure of spectral lines
- Solution: sometimes ρ(r) has *no spherical symmetry!* → electric quadrupole moment → measures the deviation from spherical symmetry

$$Q_0 \equiv \int \rho(\vec{r}\,')(3z'^2 - r'^2)\,dv$$

(x',y',z') system of the nucleus
z' is the projection of r' on the symmetry axis

$$Q_{sp}(g.s.) = \frac{I(2I-1)}{(I+1)(2I+3)} Q_0$$

Q_{sp} is a „projection" of Q_0

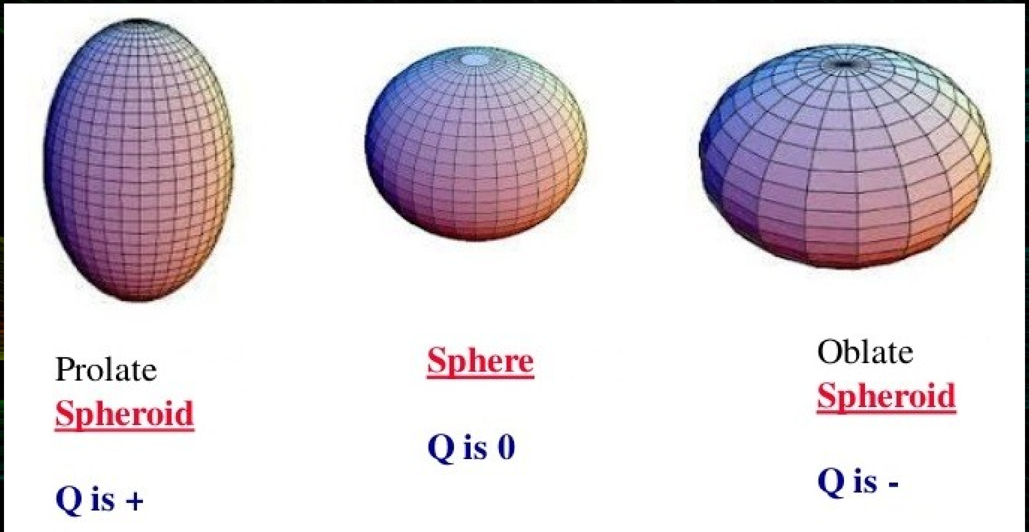
$$Q_{sp} \equiv \int \rho(\vec{r})(3z^2 - r^2)\,dv$$

spectroscopic measurement: (x,y,z) fixed in space

[Q] = Coulomb x meter^2
(often: e x cm^2) or barn

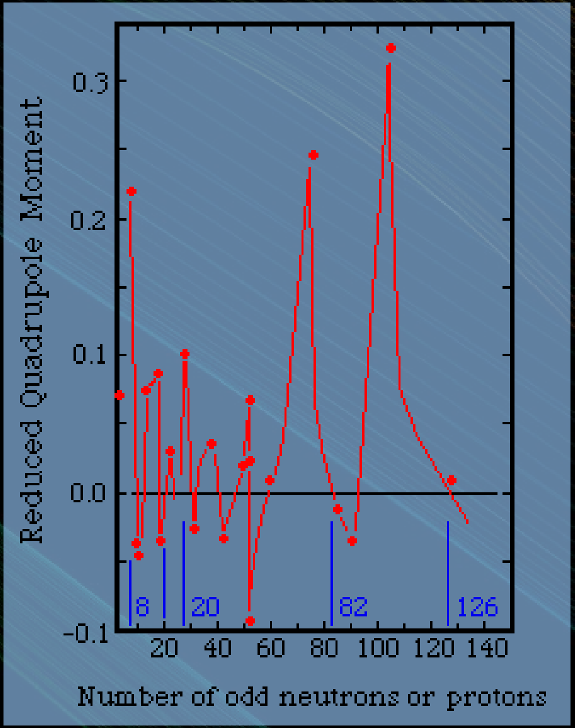
- If $Q_0 \neq 0$ then $|Q_0| > |Q_{sp}|$
- For I=0 or ½ → $Q_{sp}=0$ altough $Q_0 \neq 0$
- Q_0 can be determined *directly* by Coulomb excitation of rotational levels

Nuclear moments: electric quadrupole moment



- Deformed nuclear shape: prolate, oblate (based on Q_0)

- Results of measurements
  - magic numbers → spherical shapes
  - prolate shape is dominating for large A
  - Sometimes very strong deformation: superdeformation
  - deuteron: Q=0.00182 barn tough only 1 proton!
    - deuteron is 96% in s-state (l=0) and 4% in d-state (l=2) → reason for the magnetic moment anomaly ($\mu_d \neq \mu_p + \mu_n$)



Nuclear moments: Parity

- Quantummechanics – Schrödinger equation: success in e.g. atomic levels and alpha-decay

Hamiltonian of Schrödinger equation

$$\hat{H} = -\sum \frac{\hbar^2}{2m_i}\left(\frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + \frac{\partial^2}{\partial z_i^2}\right) + U(x_i, y_i, z_i)$$

kinetic energy

potential energy

what happens when flipping the sign of (x_i, y_i, z_i) to $(-x_i, -y_i, -z_i)$?

- Schrödinger equation has mirror symmetry → Ψ(r) has mirror symmetry as well!!

$$|\psi(x,y,z)|^2 = |\psi(-x,-y,-z)|^2$$

$$\psi(-x,-y,-z) = \exp(ia)\,\psi(x,y,z)$$

$$\psi(x) = \exp(2ia)\,\psi(x)$$

$$\exp(ia) = \pm 1$$

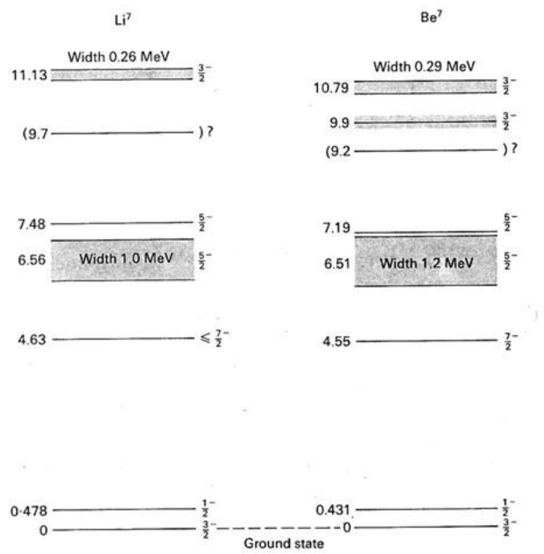
$$\psi(-x,-y,-z) = \pm\psi(x,y,z)$$

- P=+1 parity is even; P=-1 parity is odd (by definition, P=+1 for a nucleon)
- Parity of a complex system: $P_{A+B} = P_A P_B (-1)^{lA} (-1)^{lB}$ (lA,lB: quantum number of relative orbital angular momentum of A and B in the center of mass system
- Depending on the Hamiltonian, P is conserved: e.g. in strong and in EM
- At large excitation energies, overlapping states → mixed parity

Nuclear moments: Isospin

- Some light isobar nuclei (e.g. mirror nuclei) are very similiar: excitation energies, spin, parity → multiplets

Mirror Nuclei - A = 7



T=1/2 T_3=-1/2

T=1/2 T_3=+1/2

•Here we see the same two level schemes with the Coulomb energy of ~ 1.5 MeV removed.

$dE=dU_c-dm_N$

•This clearly shows that nuclear Forces are charge symmetric

i.e n-n = p-p

- if n-n=p-p=n-p (separating the effect of the Coulomb potential) then neutron = proton!
- spin formalism:

$$|\vec{T}|=T(T+1)$$

$$T_3=T, T-1, \ldots, -T$$

(M=2T+1 different value)

T: isobar-spin or isospin vector
T: isospin quantum number

- neutron and proton are doublets → $M=2$ → $T_N=1/2$ → $T_{3(n)}=-1/2$ and $T_{3(p)}=1/2$
- for a nucleus with Z,N: $T_3=(Z-N)/2$ → $T\geq|Z-N|/2$ (mostly = in g.s.)
- isospin is conserved in nuclear interactions but not in EM (γ decay) and weak!

Summary

- Historical aspects of nuclear physics: Rutherford, Cadwick
- Radii, densities
- Mass
- Spin
- Magnetic dipole moment
- Electric quadrupole moment
- Parity
- Isospin